Eng. Math (2B)

PD2 "Partial differential equation"

- 1^{st} order
- 2^{nd} order

Haromonic analysis

① fouries - series

② fouries - transform

③ LaPLice transforms

- $Q(x, y, z) = x^2 + y^2 + z^2$

fatal error

dependent

Independent

① Vector - differential operator (∇)

$$\nabla \text{ [nable] } = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$= \partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k}$$

② scalor function and vector function

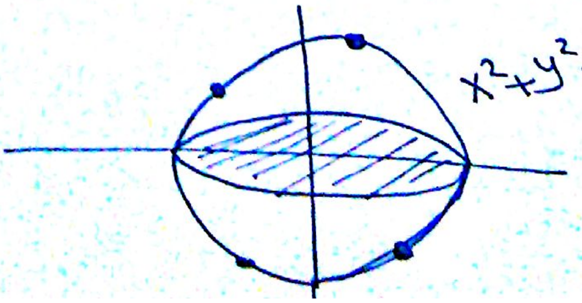
scalar function $Q = x^2 + y^2 + xy + \sin z$

داله تعتمد على موضع النقطة وقيمتها تكون ثابته عند النقطه

$Q(x, y, z) \longrightarrow$ scalar function

Sphere

$$Q(x, y, z) = x^2 + y^2 + z^2 - R^2 = 0$$



$x^2 + y^2 + z^2 = R^2$

③ vector function

$$\overline{F} = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$$

$$\overline{F} = \underbrace{(x^2+y^2+z^2)}_{F_1}\hat{i} + \underbrace{e^{xyz}}_{F_2}\hat{j} + \underbrace{x^2y}_{F_3}\hat{k}$$

(vector field) مجال اتجاهي

ex $\longrightarrow$ $\overline{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$$\overline{H} = H_x\hat{i} + H_y\hat{j} + H_z\hat{k}$$

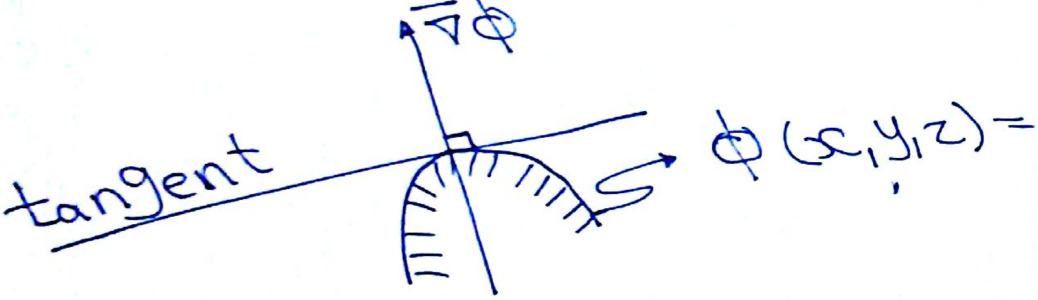
$$\rho = \rho(x,y,z) \text{ scalar}$$

الانحدار

Ⅰ Gradient of scalar function (field)

$$\text{Grad } \phi = \overline{\nabla}\phi$$

$$\overline{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$$



Ⅱ Divergence التباعد

$$\text{Div } \overline{F} = \overline{\nabla} \odot \overline{F} \neq \overline{F} \odot \overline{\nabla}$$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} = 0$$

Incompresible $\longleftarrow$ Solenoidal field

mechanical

$\neq 0$ non-solenoidal field

Compresial

III Rotation (Curl) الدوران

$$\text{Curl}(\overline{F}) = \text{Rot}\,\overline{F}$$
$$= \overline{\nabla} \times \overline{F}$$
$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

بنقيس مقدار الدوران

$$\text{Rot}\,\overline{F} = \text{Curl}\,\overline{F} = \overline{\nabla} + \overline{F} = 0$$

$\overline{F}$ is irrotational field مجال غير دوراني

$$\text{Rot}\,\overline{F} = \text{Curl}\,\overline{F} = \overline{\nabla} \times \overline{F} \neq 0$$

$\overline{F}$ is Rototional field مجال دوراني

➡ Laplacion operator (Δ)

$$\Delta = \overline{\nabla} \odot \overline{\nabla}$$
$$= (\partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k}) \odot (\partial_x \hat{i} + \partial_y \hat{j} + \partial_z \hat{k})$$
$$= \partial_{xx} + \partial_{yy} + \partial_{zz}$$
$$= \frac{\partial^2}{\partial x_2} + \frac{\partial^2}{\partial y_2} + \frac{\partial^2}{\partial z_2}$$

➡ Harmonic function دالة توافقية

$$\Delta \varphi = 0$$

φ is harmonic function

Example $\varphi = x^2 - y^2$

$$\frac{\partial^2 \varphi}{\partial x^2} = 2 , \quad \frac{\partial^2 \varphi}{\partial y^2} = -2$$

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 2 - 2 = 0$$

→ What is The PDE?

An equation which contains the unknown function and its partial deviration

$u_x + u_y = 0$

$u_x(x, y, z) = 0$

$(u_x)^2 + u_{xx} + u_{yyy} = 0$

$u_x = \frac{\partial u}{\partial x}$

$u_{xx} = \frac{\partial^2 u}{\partial x^2}$

$u_{xxx} = \frac{\partial^3 u}{\partial x^3}$

$u_{xy} = \frac{\partial^2 u}{\partial x \partial y}$

* The order of PDE :

is the highest ordered-deveration in the equation. The Degree of PDE.

The Power of highest-ordered deveration in the equation

ex ① $u_{xxx} + (\Delta u)^2 = 0$

order → 3

degree → 1

② $\frac{\partial^{3/2} u}{\partial x^{3/2}} + (u_y)^2 = 0$

order → 3/2

Degree → 1

ex The general PDE of first order in two diverations :-

$F(x, y, u, u_x, u_y) = 0$ معادلة تفاضلية جزئية من الدرجة الأولى

$(u_x)^2 + (u_y)^2 + x_u = 0$

order → 1

degree → 2

Ex The general PDE of second-order deveration in two variable (dimensions)

$$f(x, y, u, u_x, u_y, u_{xx}, u_{yy}, u_{xy}) = 0$$

$$(u_{xy})^2 + u_{xx} + u_y = 0$$

order $\longrightarrow$ 2

Degree $\longrightarrow$ 2

→ Types of 2^{nd} order PDE :-

① Linear 2^{nd} order PDE :→

إمــــا دوال في الـ x أو ثــوابــت

$$A(x,y)\,u_{xx} + B(x,y)\,u_{xy} + C(x,y)\,u_{yy} + D(x,y)\,u_x + E(x,y)\,u_y + F(x,y)\,u = G(x,y)$$

$A, B, C, D, E, F, G \Longrightarrow$ لا يعتمد أي ثابت منهم على الـ u أو تفاضلات الـ u "دوال في الـ x والـ y أو ثوابت"

$G(x,y) = 0 \longrightarrow$ Homogenous

② Semi Linear 2^{nd} order PDE

$F, G \longrightarrow$ كلاهما يعتمد على u

$$A(x,y)\,u_{xx} + B(x,y)\,u_{xy} + C(x,y)\,u_{yy} + D(x,y)\,u_x + E(x,y)\,u_y = G(x,y,u)$$

→ non-linear

$$u_{xx} + u_{yy} + u = u^2 \longrightarrow$$

$$u_{xx} + u_{yy} = u^2 - u$$

③ almost Linear 2^{nd} order PDE

$$A(x,y)\,u_{xx} + B(x,y)\,u_{xy} + C(x,y)\,u_{yy} + G(x,y,\underbrace{u, u_x, u_y}) = 0$$

→ لازم يكون الأس الكبير الـ 1

$$u_{yy} + (u_x)^2 + u = 0$$

④ Quasi-Linear PDE →

$$A(x,y,u,u_x,u_y)\,u_{xx} + B(x,y,u,u_x,u_y)\,u_{xy} + C(x,y,u,u_x,u_y)\,u_{yy} + G(x,y,u,u_x,u_y) = 0$$

$$u_x\, u_{xx} + u\, u_{yy} + u_{xy} = 0$$

$A, B, C \longrightarrow$ لاتحتوي على أي من التفاضلات العليا $(u_{xx}, u_{xy} + u_{yy})$

⑤ Non-Linear PDE $\longrightarrow$ it is not Linear

Semi-Linear
almost-linear
Quasi-Linear

$$(u_x)^2 + u = 0$$

الـ second-order المعادله الغير خطيه Non-linear PDE

اذا كانت A, B, C تحتوي على أي من التفاضلات العليا (u_{xx}, u_{xy}, u_{yy})

eg $(u_{xy})^2 + u_{xx} + u_x^2 = u$

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